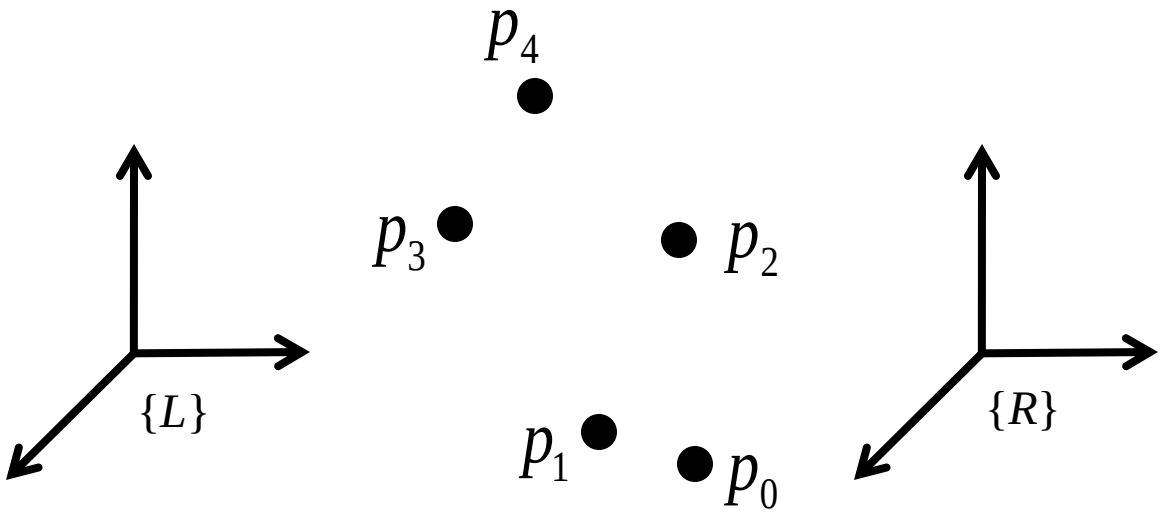
Day 05

Horn's Method

Problem Statement

- given n points measured in $\{L\}$ and $\{R\}$ estimate the transformation T_L^R (or T_R^L)



Problem Statement

- given n points measured in $\{L\}$ and $\{R\}$

$$r_{l,i} \text{ and } r_{r,i}, \; i = 1,2,...,n$$

- find the transformation

$$r_{r,i} = sRr_{l,i} + r_0$$

- residual errors

$$e_i = r_{r,i} - sRr_{l,i} - r_0$$

- find the least-squares solution

$$\sum_{i=1}^{n} \left\| e_i \right\|^2$$

1. Compute the Centroids

- compute and subtract centroids from both sets of measurements

$$\bar{r}_l = \frac{1}{n}\sum_{i=1}^{n} r_{l,i} \qquad \bar{r}_r = \frac{1}{n}\sum_{i=1}^{n} r_{r,i}$$

$$r'_{l,i} = r_{l,i} - \bar{r}_l \qquad r'_{r,i} = r_{r,i} - \bar{r}_r$$

2. Compute the Matrix M

- compute the sum of the component products for all pairs of corresponding points

$$M = \sum_{i=1}^{n} r'_{l,i} r'^{T}_{r,i} = \begin{bmatrix} S_{xx} & S_{xy} & S_{xx} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix}$$

3. Compute the Matrix N

- use the elements of M to compute N

$$N = \begin{bmatrix} S_{xx}+S_{yy}+S_{zz} & S_{yz}-S_{zy} & S_{zx}-S_{xz} & S_{xy}-S_{yx} \\ S_{yz}-S_{zy} & S_{xx}-S_{yy}-S_{zz} & S_{xy}+S_{yx} & S_{zx}+S_{xz} \\ S_{zx}-S_{xz} & S_{xy}+S_{yx} & -S_{xx}+S_{yy}-S_{zz} & S_{yz}+S_{zy} \\ S_{xy}-S_{yx} & S_{zx}-S_{xz} & S_{yz}+S_{zy} & -S_{xx}-S_{yy}+S_{zz} \end{bmatrix}$$

  - note that N is symmetric

4. Compute Eigenvectors and Eigenvalues

- compute the eigenvectors and eigenvalues of N
  - in principle, the eigenvectors and eigenvalues can be found by finding the roots of a 4th order polynomial but numeric methods are easier
    - the fact that N is symmetric allows for efficient numeric eigenvalue/vector solvers
    - in Matlab

```
% D is a diagonal matrix of eigenvalues
% columns of V are the corresponding eigenvectors
[V, D] = eig(N);
```

5. Find the Quaternion

- the quaternion representing the estimated rotation is the eigenvector corresponding to the largest positive eigenvalue
  - in Matlab

```
% y is the largest eigenvalue
% idx is the index of y in diag(D)
[y, idx] = max(diag(D));
Q = V(:, idx);
```

6. Compute the Translation

- use the centroids to compute the estimated translation
  - recall that

$$r_{r,i} = Rr_{l,i} + r_0$$

  - using the centroids

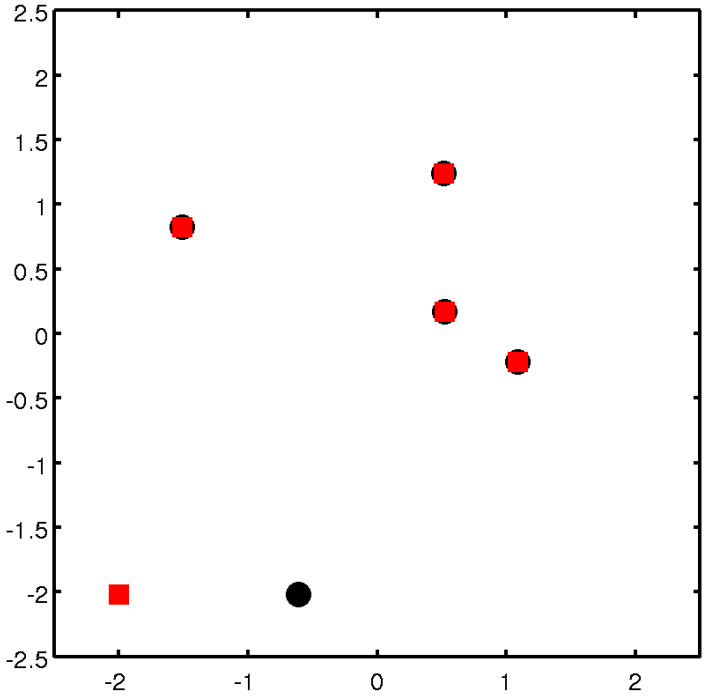
$$\bar{r}_r = R\bar{r}_l + r_0 \;\Rightarrow\; r_0 = \bar{r}_r - R\bar{r}_l$$

Issues

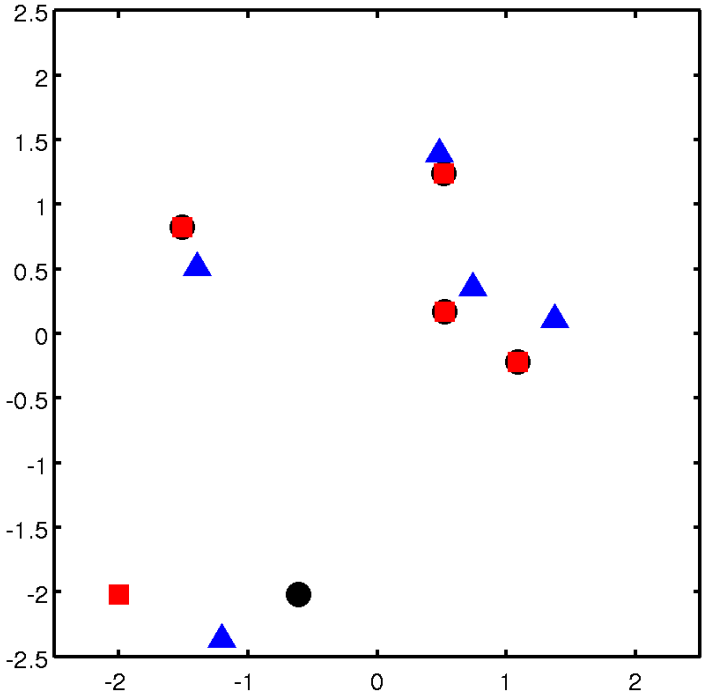
- the use of the least-squares criteria assumes
  - identically distributed noise in each point
  - isotropic noise in each point
    - more accurate (although more complicated) algorithms are available if these criteria are not met
      - Matei and Meer, IEEE PAMI, 28(10), Oct 2006

Issues

- inaccurate measurements (statistical outliers) will lead to a poor estimate of the transformation



measured



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